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# Phosphorus in Granitic Rocks of Colombia[1]

**Andrew B. Vistelius[2] and Gerardo Botero Arango[3]**

*One hundred and three samples of granitic rock taken systematically from the Andean part of Colombia reveal that the observed variation in phosphorus content reflects the variation observed for the whole-rock chemical composition. This relationship was established using a form of trend analysis termed COMTRENA and information-theory statistics.* KEY WORDS: granitic rocks, phosphorus, range number, comparative trend analysis, information-theory statistics.

## INTRODUCTION

The aim of this paper is to investigate the regularities of phosphorus distribution in granitic rocks in the Andean part of Colombia and simultaneously develop some ideas about using trend-surface analysis and the application of contingency-table methods.

Samples for the investigation were collected during 1960–1966. The treatment of data was accomplished at different times between 1968 and 1971.

## INVESTIGATING MATERIAL

The location and general composition character of granitic intrusions in Colombia are shown in Figure 1, which is taken from the geologic map of Colombia (Hubach, Radelli and Bürgl, 1962) with some modifications by G. Botero A. An examination of Figure 1 suggests a correlation between the structural location of the intrusion and rock composition. Near the Pacific coast there are quartz diorites; more acidic rocks occur in the central part of Colombia, and normal granites are located in the eastern part.

We sampled most of the granitic intrusions in the territory (Fig. 1). Information about the samples is listed in Table 1. Geographic coordinates of the sampled points were taken from the geological map (scale, 1 : 1,500,000).

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[1] Manuscript received 8 May 1972; revised 5 July 1972.
[2] Laboratory of Mathematical Geology, Academy of Sciences, Leningrad (USSR).
[3] Facultad Nacional de Minas Universidad Nacional (Colombia).

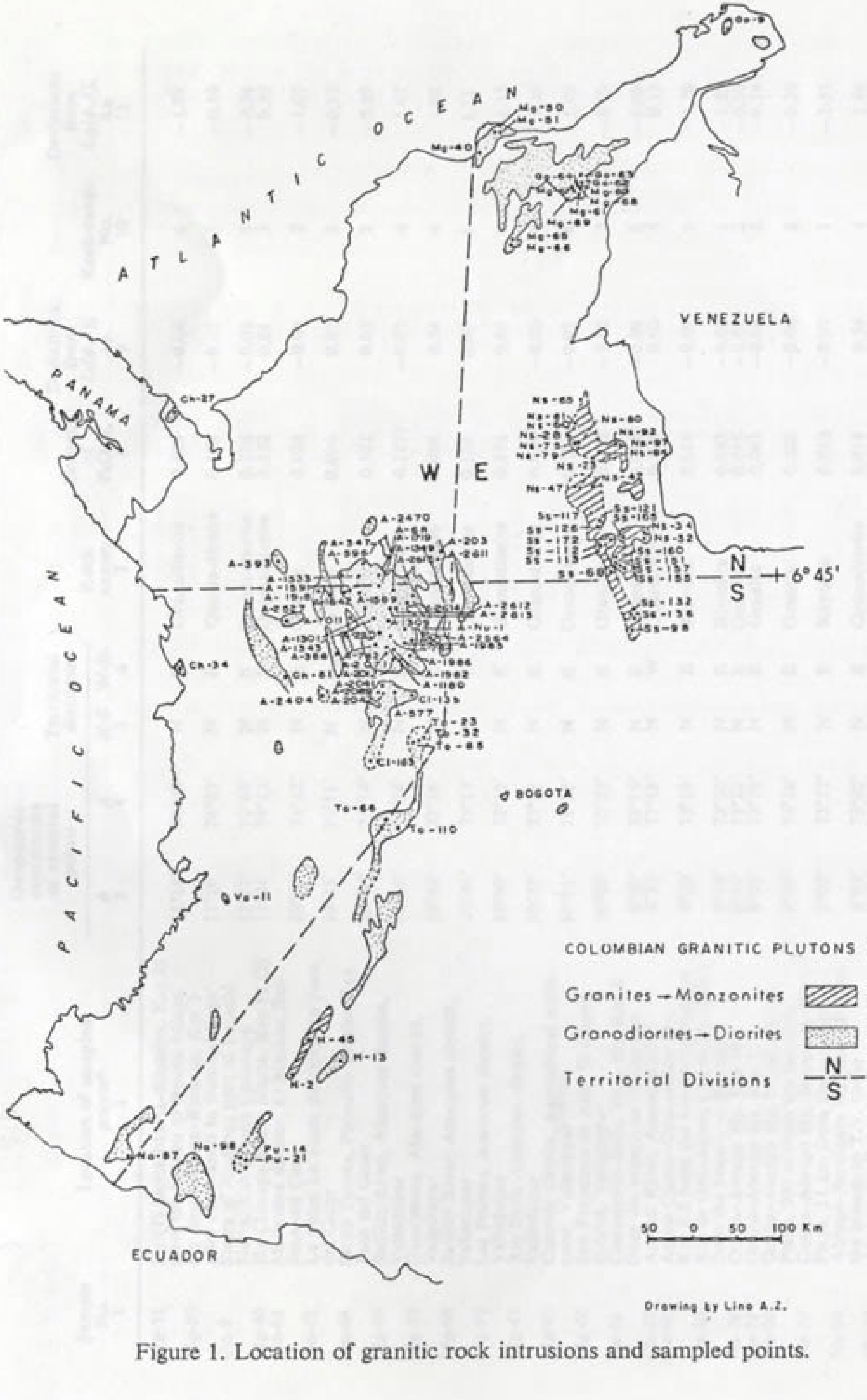


Figure 1. Location of granitic rock intrusions and sampled points.

Identification of the rock types was made by two independent investigators. One set of thin sections was identified, and all rock types were determined in Medellin. Independently, another set of thin sections from the same samples were studied in Leningrad. Most of the determinations of rock samples were in agreement. If, however, determinations were different, they were checked with additional thin sections. Content of phosphorus in samples was determined in the Laboratory of Mathematical Geology (Leningrad). Determinations were made by the colorimetric method; a ФЭК-М photocolorimeter was used for the analysis.

## GEOGRAPHIC DISTRIBUTION OF PHOSPHORUS IN COLOMBIAN GRANITIC ROCKS

The problem is as follows. We separate some character $R$, the geological meaning of which, we believe, reflects structure of the region, and determine values of $z_R(\phi,\lambda)$, where $\phi$ and $\lambda$ are geographic coordinates in $n$ observation points. Next, we take some function $L(\phi,\lambda)$, the isolines of which for some special values of its coefficients reveal a typical pattern of arrangement of $z_R$, and estimate coefficient values by approximation of $z_R(\phi,\lambda)$ by $L(\phi,\lambda)$. We term $L(\phi,\lambda)$ with estimaters of coefficients obtained from observations, a response function, and designate it by $L_R(\phi,\lambda)$.

We take the same function $L(\phi,\lambda)$ and estimate its coefficients with observed values of the character $z_0$. We make these observations on $z_0$ at the points where $z_R(\phi,\lambda)$ was obtained. The function $L(\phi,\lambda)$ with parameters estimated by observed values $z_0$ at points $z_R(\phi,\lambda)$, we designate as $L_0(\phi,\lambda)$. In this manner, we determine the pattern of isolines for $L_R(\phi,\lambda)$, the geological meaning of which is now clear. We also have the pattern of isolines of $L_0(\phi,\lambda)$. If these patterns are similar we say that there is a general reason determining the behavior of $L_R$ and $L_0$. Because the meaning of $L_R$ behavior is clear, we can use it to interprete the origin of $L_0$. Comparison of $L_R$ and $L_0$ permits us to use trend analysis in terms of a checking hypothesis. If the values of $z_R$ and $z_0$ are random variables and it is possible to compare $L_R$ and $L_0$ on the same scale, the problem is the application of an appropriate method of checking the statistical hypothesis (Lehmann, 1959). If $z_R$ and $z_0$ are not available for expression at the same scale or if one of the values of $z_R$ or $z_0$ is not a random variable, it is necessary to develop a special technique for the solution of such problems. This technique could be termed a comparative trend analysis or, simply, COMTRENA.

In our previous investigations (Vistelius and others 1969), we kept in mind the pattern of $z_R$ which was obtained from geological considerations. A similar method was developed by Romanova (1971) in her investigation of recent sand deposits of Kara-Koom desert. Here we will determine the func-

## Table 1. Observation Materials

| Sample No. 1 | Location of sampled points[a] 2 | Geographic coordinates of sampled points $\phi$ 3 | $\lambda$ 4 | Territorial divisions N-S 5 | W-E- 6 | Rock names 7 | Content of $P_2O_5$% 8 | Deviations from $L_0(\phi,\lambda)$, $\Delta_P$ 9 | Rock-range No. 10 | Deviations from $L_R(\phi,\lambda)$, $\Delta_R$ 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| Mg-51 | Hy (1) Santa Marta–Rioacha. Km 21 after the branch to Bonda-Minga | 11°16′ | 74°00′ | N | E | Granodiorite | 0.129 | −0.06 | 4 | −1.09 |
| Mg-50 | Hy Santa Marta–Rioacha. Km 3 after the branch to Bonda-Minga | 11°16′ | 74°03′ | N | E | Quartz-diorite | 0.170 | −0.02 | 5 | −0.10 |
| Go-9 | Sierra of Hudson; part of Parachi Sierra. Km 5 from Carimaya | 12°17′ | 71°40′ | N | E | Quartz-diorite | 0.186 | −0.02 | 5 | −0.36 |
| Mg-40 | Hy Ciénaga–Santa Marta. Km 12 (2) | 11°04′ | 74°13′ | N | E | Quartz-diorite | 0.172 | 0.01 | 5 | 0.52 |
| Go-63 | Malintena district, El Machín, San Juan del Cesar | 10°49′ | 73°12′ | N | E | Granite | 0.026 | −0.04 | 2 | −1.07 |
| Go-62 | La Peña, La Junta district, San Juan del Cesar | 10°45′ | 73°11′ | N | E | Granite | 0.096 | 0.02 | 2 | −0.92 |
| Go-64 | Santo Tomás, Potrerito district, San Juan del Cesar | 10°45′ | 73°14′ | N | E | Granite | 0.102 | 0.03 | 2 | −0.99 |
| Mg-60 | Badillo River, Atanques district, Valledupar | 10°43′ | 73°14′ | N | E | Granodiorite | 0.1275 | −0.02 | 4 | 1.07 |
| Mg-67 | Guasimena, Atanques district, Valledupar | 10°42′ | 73°16′ | N | E | Granodiorite | 0.488 | 0.34 | 4 | 1.05 |
| Mg-68 | Badillo River, Atanques district, Valledupar | 10°41′ | 73°13′ | N | E | Granodiorite | 0.154 | 0.00 | 4 | 1.15 |
| Mg-69 | Las Palmas, Atanques district, Valledupar | 10°40′ | 73°15′ | N | E | Granodiorite | 0.175 | 0.03 | 4 | 1.13 |
| Mg-61 | Rio Seco, Atanques district, Valledupar | 10°37′ | 73°13′ | N | E | Granodiorite | 0.095 | −0.05 | 4 | 1.26 |
| Mg-65 | Candela district, Agricultural settlement, Valledupar | 10°11′ | 73°52′ | N | E | Granodiorite | 0.132 | −0.03 | 4 | 1.09 |
| Mg-66 | San Francisco de Asís, El Copey, district, Valledupar | 10°09′ | 73°53′ | N | E | Granite | 0.082 | −0.02 | 2 | −0.89 |
| Ns-65 | Convención–Bella Vista Hy. Km 6 from the branch to El Carmen | 8°35′ | 73°15′ | N | E | Granite | 0.018 | −0.06 | 2 | −0.09 |
| Ch-27 | Acandi River, Acandi district | 8°33′ | 77°18′ | N | W | Quartz-diorite | 0.156 | 0.02 | 5 | 0.15 |
| Ns-61 | Aguas Claras–Convención Hy. Km 8.2 from the branch to Brotaré | 8°20′ | 73°19′ | N | E | Rhyolite | 0.016 | −0.07 | 1 | −1.26 |
| Ns-60 | Rio de Oro–Aguas Claras Hy. 700 m from the branch to Brotaré | 8°18′ | 73°20′ | N | E | Rhyolite | 0.053 | −0.03 | 1 | −1.29 |
| Ns-28 | Ocaña–Abrego Hy. Km 10 | 8°13′ | 73°22′ | N | E | Granite | 0.016 | −0.07 | 2 | −0.36 |
| Ns-75 | Ocaña–Abrego Hy. Km 11 | 8°09′ | 73°19′ | N | E | Granite | 0.043 | −0.04 | 2 | −0.34 |
| Ns-80 | Ocaña–Abrego Hy, branch to La Playa, 500 m from the junction | 8°09′ | 73°16′ | N | E | Granite | 0.028 | −0.06 | 2 | −0.28 |
| Ns-79 | Ocaña–Abrego Hy, Branch to La Playa, 11 km from the junction | 8°07′ | 73°17′ | N | E | Rhyolite | 0.013 | −0.07 | 1 | −1.31 |
| Ns-84 | Abrego–Sardinata Hy. Km 28 from the branch to T.V. tower | 8°03′ | 73°05′ | N | E | Granodiorite | 0.414 | 0.34 | 4 | 1.86 |
| Ns-92 | Abrego–Sardinata Hy | 8°07′ | 72°53′ | N | E | Granite | 0.0345 | −0.02 | 2 | 0.06 |

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|---|---|---|---|---|---|---|---|---|---|---|
| Ns-97 | Abrego–Sardinata Hy. 700 m before Doña Juana River | 8°06′ | 72°52′ | N | E | Granodiorite | 0.103 | 0.05 | 4 | 2.06 |
| Ns-47 | Primavera–San Alberto Hy. Km 2.4 | 7°42′ | 73°19′ | N | E | Granite | 0.015 | −0.09 | 2 | −0.54 |
| Ns-23 | Bucaramanga–Aguachica Hy. Km 2 from branch to Cáchira | 7°41′ | 73°17′ | N | E | Granite | 0.104 | 0.00 | 2 | −0.52 |
| Ns-42 | La Vega–Cáchira Hy near Los Azules gulch | 7°42′ | 73°09′ | N | E | Granite | 0.054 | −0.04 | 2 | −0.38 |
| Ss-117 | La Ceiba–La Aguada Hy | 7°26′ | 73°10′ | N | E | Granite | 0.185 | −0.02 | 2 | −0.54 |
| Ss-121 | Santa Cruz–Rionegro Hy | 7°22′ | 73°06′ | N | E | Granodiorite | 0.332 | 0.22 | 4 | 1.46 |
| Ss-126 | Sardinas–La Estufa Hy | 7°18′ | 73°07′ | N | E | Granodiorite | 0.212 | 0.10 | 4 | 1.42 |
| Ns-34 | Cácota–Chitagá Hy | 7°14′ | 72°39′ | N | E | Granite | 0.055 | 0.01 | 2 | −0.60 |
| Ss-172 | Tona–Bucaramanga Hy, La Peña River | 7°10′ | 72°58′ | N | E | Granodiorite | 0.213 | 0.11 | 4 | 1.39 |
| Ss-112 | Bucaramanga–Pamplona Hy. Km 2.5 | 7°07′ | 73°05′ | N | E | Granite | 0.054 | −0.06 | 2 | −0.68 |
| Ss-113 | Bucaramanga–Pamplona Hy. Km 5 | 7°07′ | 73°01′ | N | E | Granite | 0.439 | 0.33 | 2 | −0.66 |
| Ss-160 | Guaca–Santa Bárbara Hy | 7°00′ | 72°55′ | N | E | Granodiorite | 0.194 | 0.09 | 4 | 1.27 |
| Ss-151 | Los Curos–Guaca Hy | 6°56′ | 72°56′ | N | E | Granite | 0.050 | −0.06 | 2 | −0.78 |
| Ss-157 | Guaca–Santa Bárbara Hy, La Judia Creek | 6°55′ | 72°53′ | N | E | Granite | 0.0905 | −0.02 | 2 | −0.80 |
| Ss-155 | Guaca–Santa Bárbara Hy, near Santa Bárbara | 6°54′ | 72°52′ | N | E | Granodiorite | 0.088 | −0.02 | 4 | 1.18 |
| Ss-68 | Bucaramanga–San Gil Hy, Pescadero bridge | 6°51′ | 72°59′ | N | E | Granite | 0.085 | −0.04 | 2 | −0.83 |
| Ss-165 | Berlin Baraya Hy. Km 2.1 | 7°11′ | 72°51′ | N | E | Granodiorite | 0.012 | −0.07 | 4 | 1.41 |
| Ns-32 | Chitagá Hy. 1.5 km before branch to Cácota | 7°11′ | 72°40′ | N | E | Granite | 0.071 | 0.02 | 2 | −0.65 |
| A-2470 | Puerto Antioquia–Valdivia Hy. Km 11 | 7°24′ | 75°22′ | N | W | Granite | 0.091 | −0.05 | 2 | −1.84 |
| A-203 | Segovia–Planta Nueva Hy. Km 13 | 7°05′ | 74°41′ | N | W | Quartz-diorite | 0.166 | 0.06 | 5 | 0.92 |
| A-347 | Yarumal–Valdivia Hy. Km 8 | 6°59′ | 75°27′ | N | W | Quartz-diorite | 0.159 | 0.01 | 5 | 0.87 |
| A-393 | Medellín–Turbo Hy, Quiparadó River | 7°02′ | 76°18′ | N | W | Monzonite | 0.432 | 0.16 | 3 | −0.56 |
| A-2611 | Yalí–Remedios Hy | 7°01′ | 74°42′ | N | W | Granodiorite | 0.169 | 0.06 | 4 | −0.12 |
| A-596 | San José–Ituango road, near San José | 6°53′ | 75°40′ | N | W | Quartz-diorite | 0.163 | −0.01 | 5 | 0.91 |
| A-68 | Angostura–Yarumal Hy. Km 1 | 6°51′ | 75°17′ | N | W | Granodiorite | 0.146 | 0.01 | 4 | −0.28 |
| A-1533 | Los Llanos–San Andrés Hy. Km 13 | 6°51′ | 75°30′ | N | W | Granodiorite | 0.153 | 0.00 | 4 | −0.20 |
| A-1591 | El Chaquiro–Aragón Hy. Km 1 | 6°43′ | 75°35′ | S | W | Quartz-diorite | 0.159 | −0.00 | 5 | 0.74 |
| A-1918 | Buriticá village | 6°43′ | 75°54′ | S | W | Quartz-diorite | 0.044 | −0.16 | 5 | 0.93 |
| A-1719 | Tenche tunnel, Carolina district | 6°45′ | 75°18′ | N | W | Granodioritee | 0.129 | −0.00 | 4 | −0.34 |
| A-1349 | Porcecito–Amalfi Hy. Km 55 | 6°42′ | 75°09′ | S | W | Granodiorite | 0.114 | −0.01 | 4 | −0.38 |
| A-1642 | Belmira–El Yerbal Hy. Km 3 | 6°41′ | 75°39′ | S | W | Quartz-diorite | 0.141 | −0.03 | 5 | 0.75 |
| A-1589 | El Chaquiro–Aragón Hy. Km 7 | 6°41′ | 75°30′ | S | W | Quartz-diorite | 0.186 | 0.03 | 5 | 0.68 |
| A-2615 | Yolombó–Yalí Hy. Km 37 | 6°39′ | 74°58′ | S | W | Granodiorite | 0.102 | −0.02 | 4 | −0.38 |
| A-2614 | Sofía–Yolombó Hy. Km 8.9 | 6°34′ | 75°01′ | S | W | Granodiorite | 0.126 | 0.01 | 4 | −0.42 |
| A-1301 | Don Matías–Santa Rosa Hy, Riogrande | 6°33′ | 75°25′ | S | W | Granodiorite | 0.152 | 0.01 | 4 | −0.42 |
| A-2613 | La Quiebra–Cisneros Hy | 6°32′ | 75°09′ | S | W | Granodiorite | 0.182 | 0.06 | 4 | −0.46 |
| A-2612 | Cisneros–La Quiebra Hy. Km 7 | 6°32′ | 75°08′ | S | W | Granodiorite | 0.123 | −0.00 | 4 | −0.45 |
| A-2527 | Urrao–Caicedo Hy, Noque Creek | 6°25′ | 76°13′ | S | W | Granodiorite | 0.341 | 0.09 | 4 | −0.06 |
| A-1011 | Medellín–San Pedro Hy, El Hato Creek | 6°26′ | 75°35′ | S | W | Quartz-diorite | 0.124 | −0.04 | 5 | 0.57 |
| A-1343 | Barbosa–Amalfii Hy. Km 11 | 6°25′ | 75°21′ | S | W | Quartz-diorite | 0.156 | 0.02 | 5 | 0.50 |
| A-1309 | Concepción village, Antioquia | 6°24′ | 75°12′ | S | W | Quartz-diorite | 0.124 | −0.01 | 5 | 0.49 |
| A-290 | El Peñol–San Vicente road, El Marial | 6°18′ | 75°17′ | S | W | Granodiorite | 0.136 | 0.00 | 4 | −0.55 |

**Table 1. Continued**

| Sample No. 1 | Location of sampled points[a] 2 | Geographic coordinates of sampled points $\phi$ 3 | $\lambda$ 4 | Territorial divisions N-S 5 | W-E- 6 | Rock names 7 | Content of $P_2O_5$,% 8 | Deviations from $L_0(\phi,\lambda)$, $\Delta_P$ 9 | Rock-range No. 10 | Deviations from $L_R(\phi,\lambda)$, $\Delta_R$ 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| A-Nu-1 | Peñol–San Rafael Hy, Bizcocho River | 6°19′ | 75°02′ | S | W | Quartz-diorite | 0.131 | 0.01 | 5 | 0.50 |
| A-2564 | San Carlos–Nare Hy. Km 26 | 6°17′ | 74°51′ | S | W | Quartz-diorite | 0.131 | 0.01 | 5 | 0.57 |
| A-784 | Peñol de Guatapé, Guatapé district | 6°15′ | 75°12′ | S | W | Quartz-diorite | 0.140 | 0.01 | 5 | 0.44 |
| A-30 | San Carlos–Granada Hy. Km 2 | 6°12′ | 74°58′ | S | W | Granodiorite | 0.138 | 0.01 | 4 | −0.50 |
| A-1985 | Granada–San Carlos Hy, branch to San Luis | 6°11′ | 75°02′ | S | W | Granodiorite | 0.092 | −0.04 | 4 | −0.53 |
| A-782 | Marinilla–El Peñol Hy. Km 13 | 6°10′ | 75°20′ | S | W | Quartz-diorite | 0.206 | 0.07 | 5 | 0.41 |
| A-1180 | El Santuario–El Peñol road, Bodegas Creek | 6°09′ | 75°13′ | S | W | Quartz-diorite | 0.1495 | 0.02 | 5 | 0.42 |
| A-1982 | Granada–San Luis Hy. Km 2.5 | 6°07′ | 75°10′ | S | W | Granodiorite | 0.141 | 0.01 | 4 | −0.58 |
| A-2071 | Santuario–Cocorná road, Marimonda Creek | 6°05′ | 75°13′ | S | W | Quartz-diorite | 0.140 | 0.00 | 5 | 0.40 |
| A-386 | Angelópolis railway station | 6°05′ | 75°41′ | S | W | Granodiorite | 0.102 | −0.07 | 4 | −0.55 |
| A-1986 | San Luis–Granada road. Km 1.5 | 6°04′ | 74°59′ | S | W | Granodiorite | 0.1196 | −0.01 | 4 | −0.52 |
| A-2012 | Cocorná–San Francisco Hy. Km 8 | 6°01′ | 75°10′ | S | W | Quartz-diorite | 0.170 | 0.04 | 5 | 0.41 |
| Ch-34 | Utria bay, Pacific coast | 5°58′ | 77°18′ | S | W | Granodiorite | 0.140 | −0.08 | 4 | −0.21 |
| Ch-51 | Bolivar–Quibdó Hy. Km 16 | 5°51′ | 76°14′ | S | W | Granodiorite | 0.151 | −0.09 | 4 | −0.35 |
| A-2041 | Medellín–Sonsón Hy, Tasajo river | 5°46′ | 75°16′ | S | W | Quartz-diorite | 0.099 | −0.04 | 5 | 0.37 |
| Ss-132 | San Joaquín–Onzaga Hy, Santa Ana village | 6°24′ | 72°48′ | S | E | Granodiorite | 0.089 | −0.04 | 4 | 0.67 |
| Ss-136 | San Joaquín–Onzaga Hy, 1.5 km east of Guayabal | 6°23′ | 72°46′ | S | E | Granodiorite | 0.181 | 0.06 | 4 | 0.61 |
| Ss-98 | Central Hy–Onzaga Hy. Km 8 | 6°15′ | 72°47′ | S | E | Granite | 0.051 | −0.09 | 2 | −1.51 |
| A-2089 | Sonsón–Los Medios Hy. Km 10.5 | 5°42′ | 75°18′ | S | W | Quartz-diorite | 0.258 | 0.12 | 5 | 0.36 |
| A-2042 | Sonsón–Nariño Hy, Páramo de Sonsón | 5°41′ | 75°16′ | S | W | Quartz-diorite | 0.1196 | −0.02 | 5 | 0.37 |
| A-577 | Sonsón–Dorada Hy. Km 44 | 5°41′ | 75°09′ | S | W | Quartz-diorite | 0.262 | 0.12 | 5 | 0.42 |
| Cl-135 | Dorada–Florencia Hy. Km 43.5 | 5°39′ | 74°58′ | S | W | Quartz-diorite | 0.097 | −0.04 | 5 | 0.53 |
| A-2404 | Indes town, Antioquia | 5°39′ | 75°53′ | S | W | Granodiorite | 0.307 | 0.12 | 4 | −0.60 |
| To-23 | 2 km east of Mariquita, Tolima | 5°13′ | 74°54′ | S | W | Granodiorite | 0.116 | −0.03 | 5 | 0.73 |
| To-32 | Mariquita–Gualí River bridge | 5°12′ | 74°56′ | S | W | Quartz-diorite | 0.091 | −0.06 | 5 | 0.73 |
| To-85 | Honda–Fresno Hy. Km 39 | 5°11′ | 74°58′ | S | W | Quartz-diorite | 0.242 | 0.09 | 5 | 0.71 |
| Cl-163 | Manizales–Fresno Hy. Km 32 | 5°01′ | 75°24′ | S | W | Quartz-diorite | 0.167 | 0.02 | 5 | 0.51 |
| To-66 | Ibagué–Cajamarca Hy. Km 13 | 4°25′ | 75°16′ | S | W | Granite | 0.025 | −0.12 | 2 | −2.01 |
| To-110 | El Cobre Creek, Payandé, Tolima | 4°19′ | 75°10′ | S | E | Quartz-diorite | 0.223 | 0.07 | 5 | 1.16 |
| Va-11 | Anchicayá River, power plant | 3°27′ | 76°42′ | S | W | Granodiorite | 0.315 | 0.10 | 4 | 0.08 |
| H-45 | Popayán–Neiva Hy, Pescado River | 2°14′ | 76°05′ | S | E | Granite | 0.021 | −0 08 | 2 | −0.98 |
| H-13 | Altamira–Guadalupe Hy, Suaza River | 2°01′ | 75°47′ | S | E | Granodiorite | 0.225 | 0.13 | 4 | 1.44 |
| H-2 | Pitalito–San Agustín Hy, Sombrerillo River | 1°53′ | 76°15′ | S | E | Granite | 0.0195 | −0.08 | 2 | −1.05 |
| Na-87 | Pasto–Tumaco Hy. Km 121 | 1°08′ | 77°43′ | S | E | Quartz-diorite | 0.191 | 0.02 | 5 | 0.06 |
| Na-98 | Pasto–Los Alisales Hy. Km 36 | 1°05′ | 77°14′ | S | E | Quartz-diorite | 0.208 | 0.05 | 5 | 0.47 |
| Pu-14 | Pasto–Mocoa Hy. Km 100 | 1°04′ | 76°46′ | S | E | Granodiorite | 0.126 | 0.02 | 4 | 0.01 |
| Pu-21 | Pasto–Mocoa Hy. Km 113 | 1°04′ | 76°44′ | S | E | Granodiorite | 0.101 | 0.00 | 4 | 0.06 |

tion $L_R$ $(\phi,\lambda)$ by a formal operation. However, we do not know by formal operations how to compare the pattern of an artificially introduced function $L_R$ $(\phi,\lambda)$ with the pattern of $L_0$ $(\phi,\lambda)$ obtained by observation of phosphorus content. Therefore, we will compare $L_R$ and $L_0$ patterns visually, but in the future this method should be developed on a formal basis.

## DEFINITION OF $L(\phi,\lambda)$ AND ALGORITHMS USED

Using COMTRENA requires a definition of $L$ $(\phi,\lambda)$. At the moment we include three requirements for this function. The function should be able to outline the main geological features for the behavior of $z_i$ values. In other words, the variability of the function over the study area should be rather high. The function should be investigated mathematically as thoroughly as possible. Mathematical properties of the function should be well known. The values of $z_R$ and $z_0$ in our investigations are only positive or equal to zero. Thus, the response-function values should be only positive or equal to zero.

If we investigate different functions, we find that the polynomial approximation log values of $z_R$ and $z_0$ is such a function. Really, variability of the polynomial is sufficient for describing geometrical patterns of geological ideas. The polynomial also is investigated mathematically rather thoroughly. If we approximate with polynomial log values, and later return to natural figures from logarithms, we exclude negative values because our function will work only with positive numbers. Lastly, it is obvious that the observed surface of $z_i$ values is some solution of an stochastic differentia lequation which we are not able to determine as yet. But in many instances differential equations have exponential functions as their solution. Thus an exponential function such as $L$ $(\phi,\lambda)$ is convenient from a general point of view, and there is reason to believe that, at the moment,

$$L(\phi,\lambda) = \exp P_m(\phi,\lambda) \tag{1}$$

is sufficient for our purpose. Here $P$ is a polynomial of order $m$ with geographic coordinates $\phi$ and $\lambda$. Numerous programs for computation of polynomial trend surfaces exist now which can be used for computation of our $L$ $(\phi,\lambda)$, with an additional subroutine for determining log values. Such subroutines exist at all computer centers.

Calculation of estimaters of coefficients of the polynomial in eq (1) can be obtained by different methods. Any method is convenient because the important problem is determining the pattern of isolines and not the behavior of variance, etc. It is important only that in all instances the pattern of isolines should express the behavior of $z_i$ values but not the result of computer work. The best method in this instance is the one which guarantees the smallest change of estimaters of polynomial coefficients with a small change in the coordinates $\phi$ and $\lambda$. In other words, it is desirable that small changes of the sampling net reflect only small changes in behavior of the isoline pattern.

We used a least-square procedure based on Bauer's investigations (1963, 1965) for estimation of coefficients.

The use of a least-square operation for determining a generalized system of linear equations $x$ minimizes the Euclidean norm of a difference

$$\ell - Ax$$

where $\ell$ is a vector of observed values ($z_R$ and $z_0$ in our situation) and $A$ is a matrix of the system composed of values of some function of $\phi$ and $\lambda$ in observation points. Matrix $A$ usually has more rows than columns.

The solution for $x$ can be obtained from

$$A^T Ax = A^T \ell \qquad (2)$$

where $T$ is a sign of transposition.

The main difficulty in solving eq (2) is connected with inverting matrix $A^T A$ because precision is lost in the inversion. This requires a special property in matrix $A^T A$ which is termed conditionality. One of the measures of a conditionality is the Todd number, which is a ratio of the largest eigenvalue of matrix $A^T A$ to the smallest eigenvalue. Investigation of conditionality of the matrix requires a square matrix $A^T A$. Obtaining a square matrix from a rectangular matrix $A$ also results in the loss of precision. Computations for this paper were based on Bauer's (1963) procedure for improving the conditionality of matrix $A^T A$ by optimal scaling. In this situation we were able to evaluate results of improvement by computing the Todd numbers before and after optimal scaling. The program for this method was prepared by D. V. Sharkov (we will call it algorithm $B_1$).

In a recent investigation, Bauer (1965) suggested an algorithm which avoids using a square matrix. There is a special determination of a new orthogonal base in the space-determined independent columns of matrix $A$. In this situation eq (2) is replaced by

$$v^T Ax = v^T \ell$$

where $v$ is a matrix, the columns of which compose an orthogonal base of columns for matrix $A$. The matrix $v^T A$ has better conditionality than matrix $A^T A$. The program for this situation is published (Graunov and Romanova, 1971), and the algorithm we will term $B_2$. After these preliminary considerations, we are able now to give a clear picture of results in applying COMTRENA to our problem.

## REGIONAL VARIABILITY OF ROCK COMPOSITION AND PHOSPHORUS CONCENTRATIONS

It was mentioned previously that computations were made with the algorithm using $A^T A$ with optimal scaling. Todd numbers obtained in the process are listed in Table 2.

An analysis of the Table 2 indicates that third-degree polynomial trend surfaces are stable in conditionality of the matrix, and small changes in coordinate values do not change the isoline pattern. Polynomials of the fourth order and higher show the matrix to be ill-conditioned, and precision of the computations can influence the isoline pattern. It is necessary to keep in mind also that the real configuration is more stable than the configuration obtained from Todd numbers. Todd numbers indicate stability of a whole functional, but we are interested only in stability of its geometry in the region of approximation.

We have discussed that trend-surface analysis can be developed from a heuristic level to a mathematical level if we introduce $L_R(\phi,\lambda)$ and determine values of $\varXiR$. In our situation the character $\Xi_R$ should be separated so that its influence on the phosphorus concentration can be checked. Because we are interested in elucidating the influence of regional changes of rock composition on the trend in phosphorus concentrations, we should determine the values of $\Xi_R$ as some character of the rock composition. Because we had no modal or chemical analyses of the samples, we used rock names for determination of $\Xi_R$. It was indicated that the identification of rock type was made carefully. When all samples were identified, we gave each rock type a range number associated with the content of silica in the rock. Thus we obtained the following range numbers

| Rock: | rhyolite | granite | monzonite | granodiorite | quartz-diorite |
|---|---|---|---|---|---|
| range number: | 1 | 2 | 3 | 4 | 5 |

**Table 2. Todd Numbers for Matrices $A^T A$ of Normal Systems of Polynomials from First to Fifth Order for Accepted Sampling Net**

| Order of the polynomial corresponding $A^T A$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Todd number before optimal scaling | $1.8 \times 10^3$ | $4.8 \times 10^6$ | $1.3 \times 10^{10}$ | $1.2 \times 10^{12}$ | $9.7 \times 10^{13}$ |
| Todd number after optimal scaling | $1.8 \times 10^1$ | $1.9 \times 10^1$ | $4.9 \times 10^2$ | $2.8 \times 10^5$ | $2.6 \times 10^6$ |

The function $L_R$ $(\phi,\lambda)$ was calculated for $\varepsilon_R$ expressed in range numbers. The function $L_R$ $(\phi,\lambda)$ was calculated as a polynomial without log transformation of $\varepsilon_R$ because range numbers were determined arbitrarily, and they can be substituted by any monotone function including log transformation. Values of $L_0$ $(\phi,\lambda)$ were calculated as an exponential function (1). We evaluated the most general tendency in changes of $\varepsilon_R$ and $\varepsilon_0$. For this purpose we computed estimaters of the function coefficients

$$L_R(\phi,\lambda) = a_0^{(R)} + a_1^{(R)}\phi + a_2^{(R)}\lambda$$

and

$$L_0(\phi,\lambda) = \exp\,(a_0^{(0)} + a_1^{(0)}\phi + a_2^{(0)}\lambda)$$

Isoline functions $L_R$ $(\phi,\lambda)$ and $L_0$ $(\phi,\lambda)$ are plotted on Figures 2 and 3. It is clear that the isoline patterns for both functions are coincident. Geological meaning of the patterns is clear. The composition of the rocks and phosphorus content in these rocks are changing parallel to one another from the boundary of the Guiana Shield to the coast of the Pacific Ocean. The most acidic rocks are located near the shield and are poor in phosphorus. Near the Pacific shore, the rocks are poor in silica and rich in phosphorus. The strike of rock-range number isolines and phosphorus concentration are close to the strike of the mountain system.

To reveal detail of rock-range number and phosphorus concentrations over the territory requires a trend surface of higher order. Because the fourth-order polynomial trend surface showed only slight instability, we used this polynomial. We assume that it can be stable in the restricted region of approximation, and estimated the coefficients of

$$L(\phi,\lambda) = \exp\, P_4(\phi,\lambda) \tag{3}$$

Figure 4 indicates the isoline pattern for rock-range numbers. Figure 5 gives the isoline pattern of phosphorus concentrations. Because the sampling net has generated a rather ill-conditioned matrix we repeated the calculations twice. The thick lines are isolines of eq (3) computed by Sharkov's program (algorithm $B_1$). The dotted lines are the results of approximation of the same function with coefficients estimated by the program based on a rectangular matrix (Graunov and Romanova, 1971). It is clear that the isolines obtained by different methods are similar. This indicates that the function is rather stable within the region of approximation and the picture revealed is real.

Comparison of results in fitting $\varepsilon_R$ and $\varepsilon_0$ by eq (3) shown in Figures 4 and 5 indicates that there is some similarity in both patterns. Thus, in northern Colombia there is a simultaneous increase in the concentration of phosphorus and decrease in content of silica in the rocks. There is a dominance of acidic rocks from the southern part of the boundary with Venezuela to Panama,

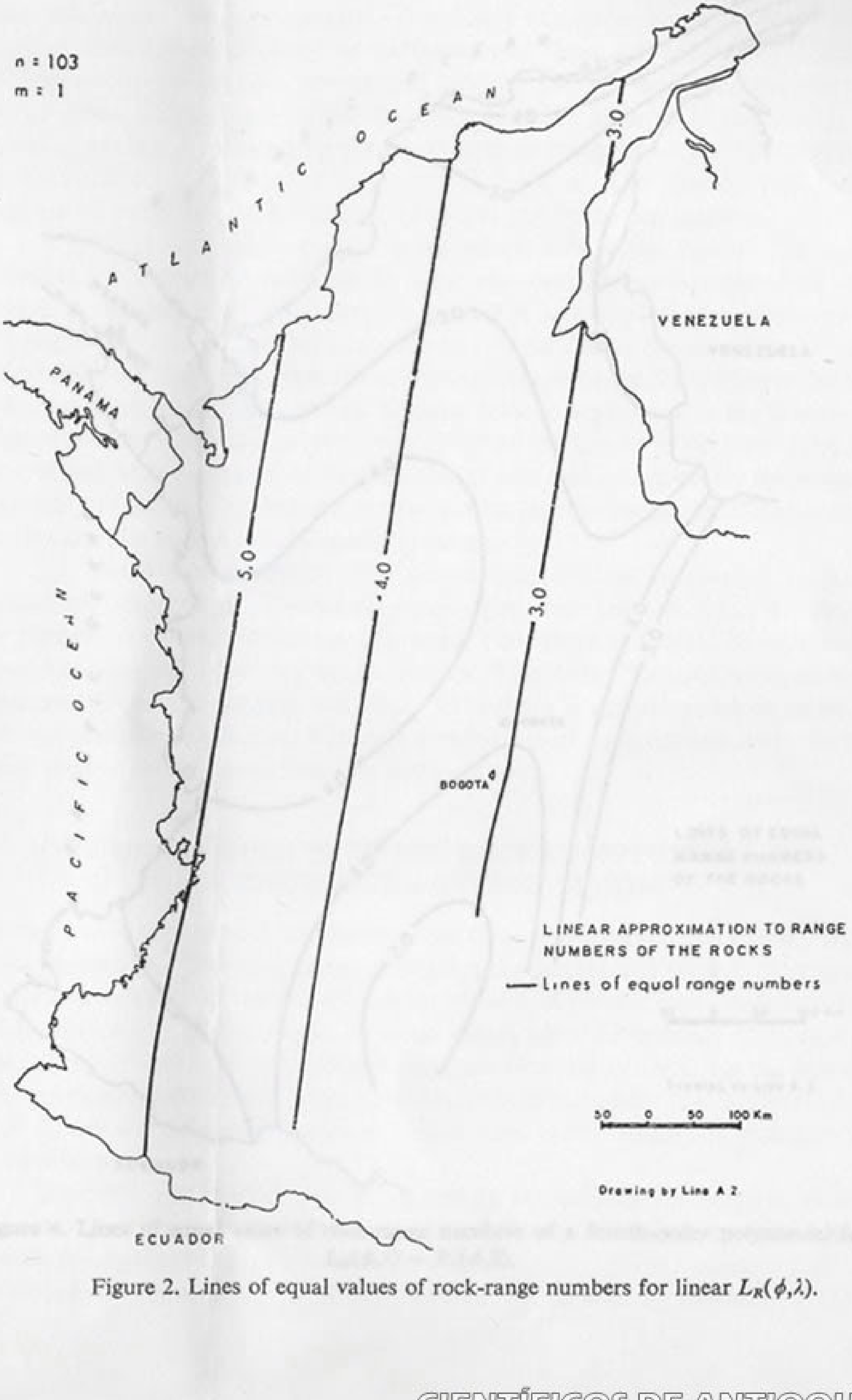


Figure 2. Lines of equal values of rock-range numbers for linear $L_R(\phi,\lambda)$.

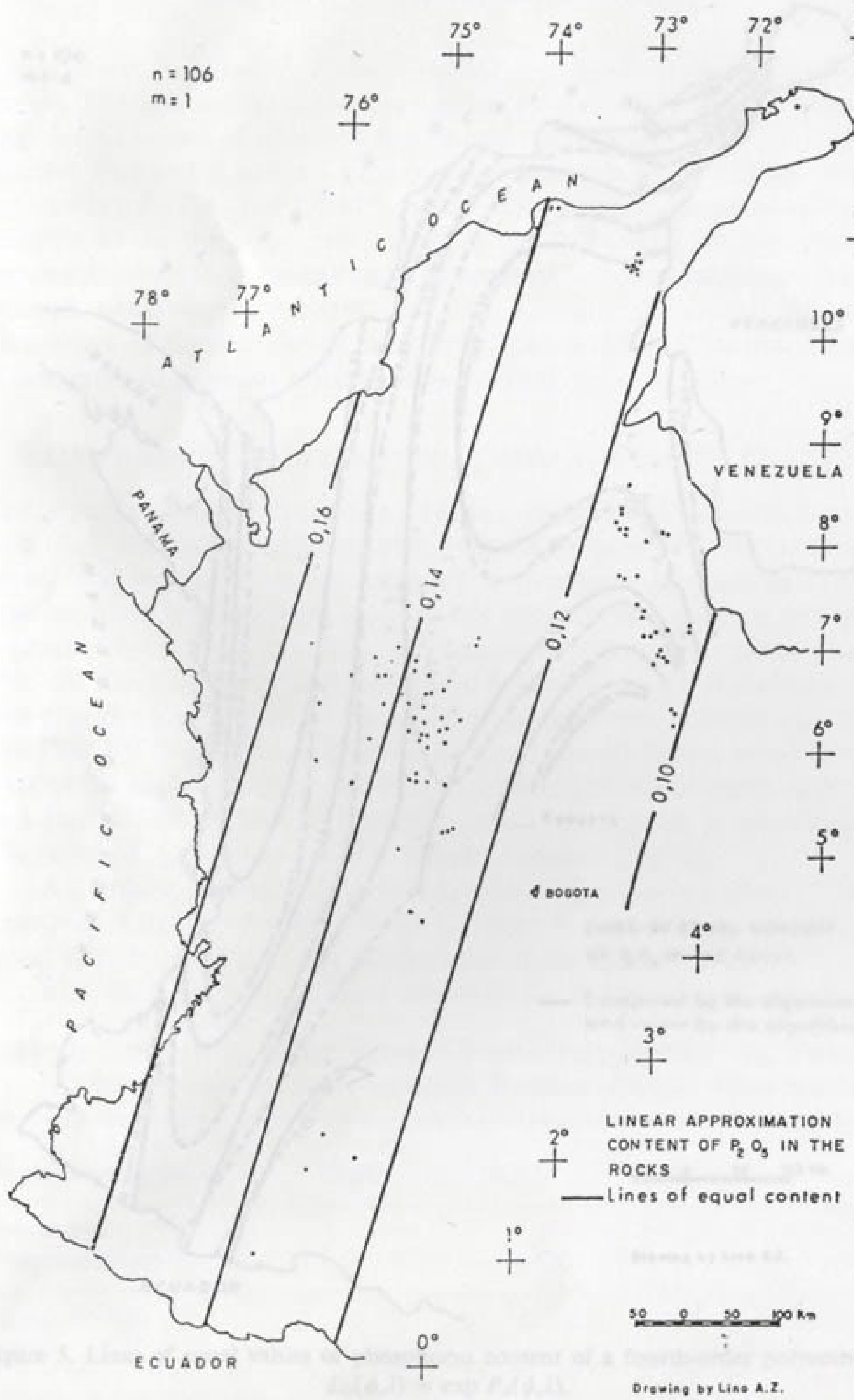


Figure 3. Lines of equal values of phosphorus content for linear $L_0(\phi,\lambda)$.

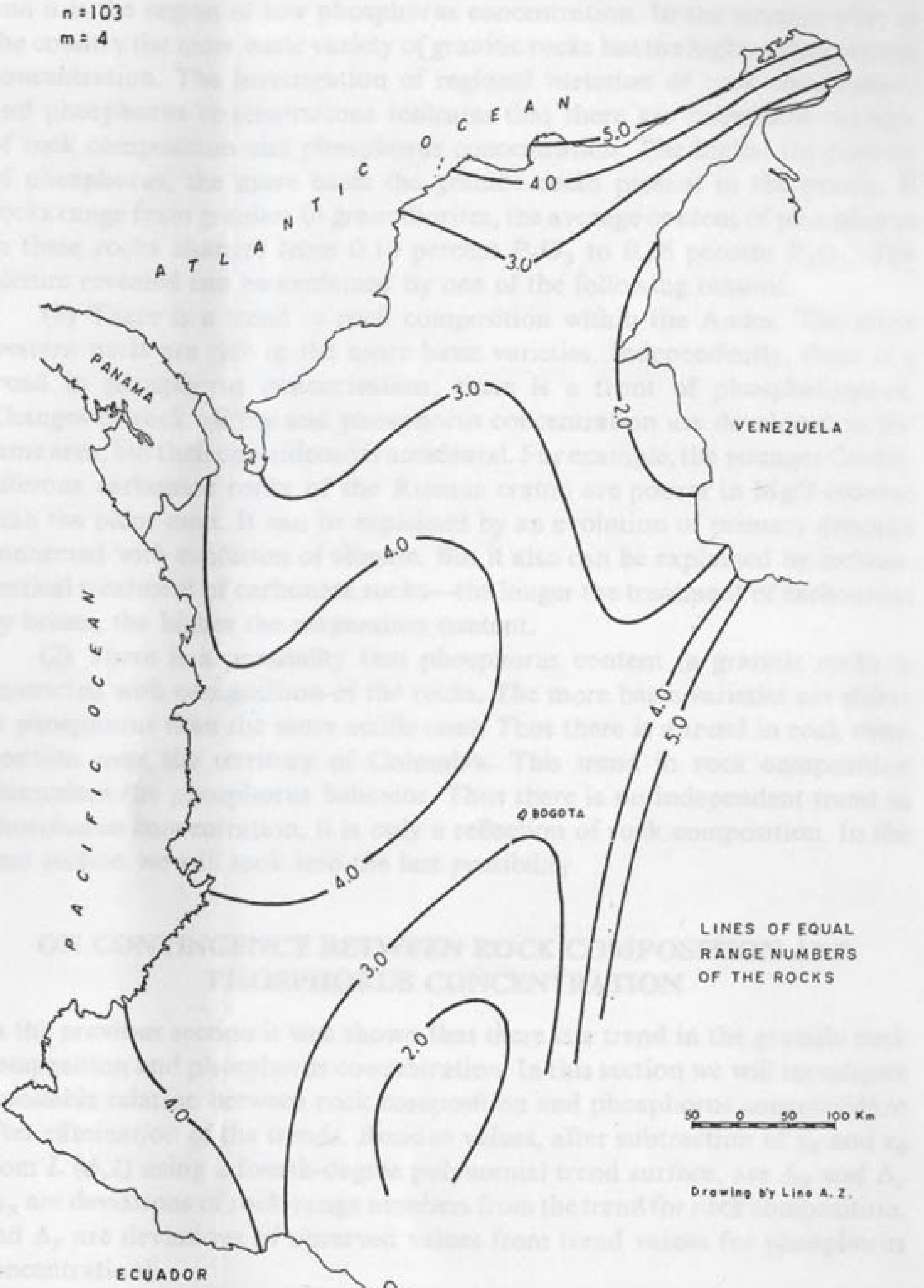


Figure 4. Lines of equal value of rock-range numbers of a fourth-order polynomial for $L_R(\phi,\lambda) = P_4(\phi,\lambda)$.

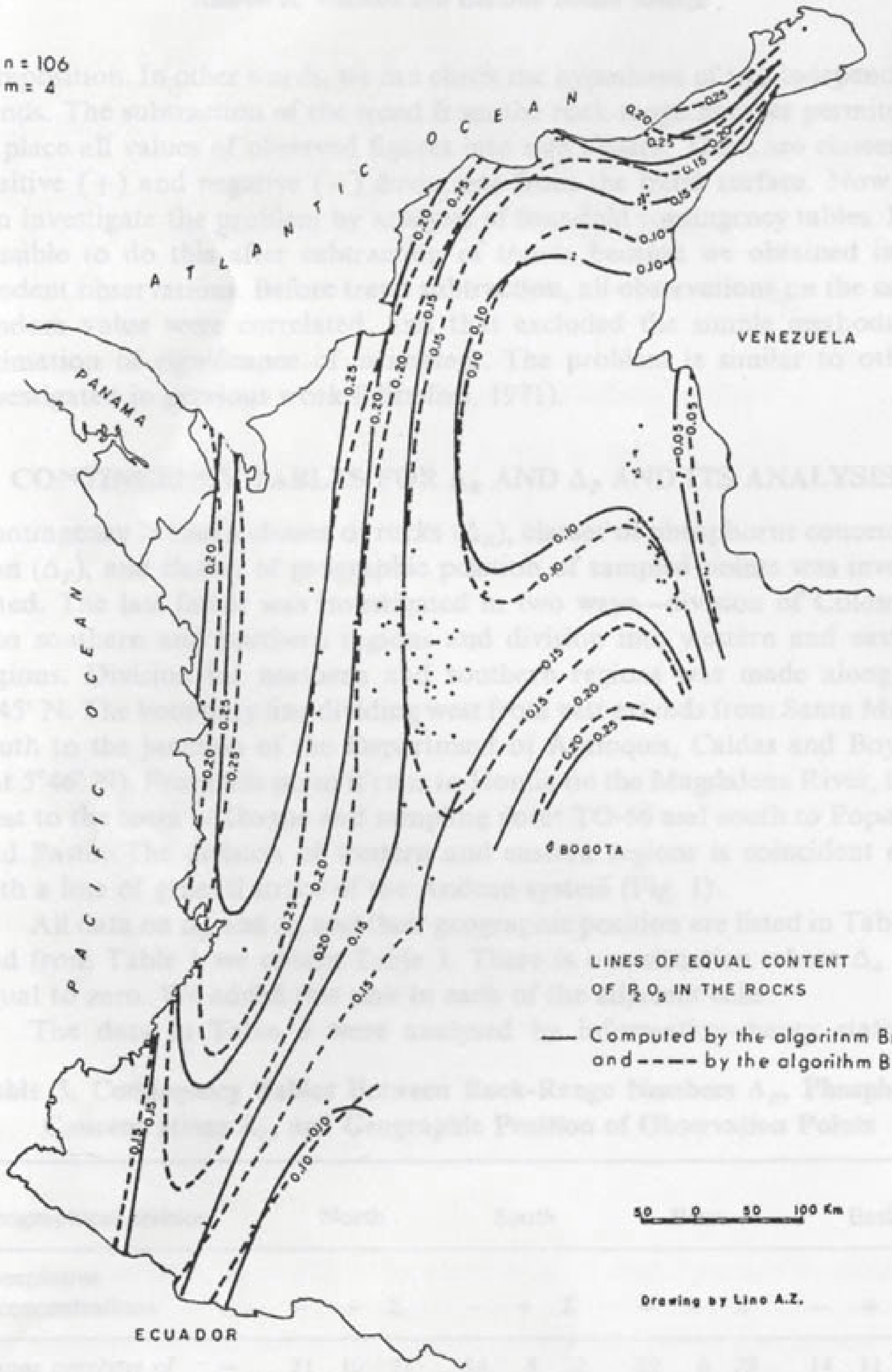


Figure 5. Lines of equal values of phosphorus content of a fourth-order polynomial for $L_0(\phi,\lambda) = \exp P_4(\phi,\lambda)$.

and it is the region of low phosphorus concentration. In the western part of the country the more basic variety of granitic rocks has the highest phosphorus concentration. The investigation of regional variation of rock composition and phosphorus concentrations indicates that there are coincident changes of rock composition and phosphorus concentration. The higher the content of phosphorus, the more basic the granitic rocks present in the region. If rocks range from granites to granodiorites, the average content of phosphorus in these rocks changes from 0.10 percent $P_2O_5$ to 0.16 percent $P_2O_5$. The picture revealed can be explained by one of the following reasons.

(1) There is a trend in rock composition within the Andes. The more western parts are rich in the more basic varieties. Independently, there is a trend in phosphorus concentration; there is a front of phosphatization. Changes of rock acidity and phosphorus concentration are developed in the same area, but their coincidence is accidental. For example, the younger Carboniferous carbonate rocks of the Russian craton are poorer in MgO content than the older ones. It can be explained by an evolution of primary deposits connected with evolution of climate. But it also can be explained by metasomatical treatment of carbonate rocks—the longer the treatment of carbonates by brines, the higher the magnesium content.

(2) There is a possibility that phosphorus content in granitic rocks is connected with composition of the rocks. The more basic varieties are richer in phosphorus than the more acidic ones. Thus there is a trend in rock composition over the territory of Colombia. This trend in rock composition determines the phosphorus behavior. Thus there is no independent trend in phosphorus concentration, it is only a reflection of rock composition. In the next section we will look into the last possibility.

## ON CONTINGENCY BETWEEN ROCK COMPOSITION AND PHOSPHORUS CONCENTRATION

In the previous section it was shown that there is a trend in the granitic rock composition and phosphorus concentration. In this section we will investigate a possible relation between rock composition and phosphorus concentration after elimination of the trends. Residue values, after subtraction of $z_R$ and $z_0$ from $L\ (\phi,\lambda)$ using a fourth-degree polynomial trend surface, are $\Delta_R$ and $\Delta_P$ ($\Delta_R$ are deviations of rock-range numbers from the trend for rock composition, and $\Delta_P$ are deviations of observed values from trend values for phosphorus concentration).

After elimination of trends for $z_R$ and $z_0$, we obtained two random values characterized by independent observations. Now, if deviations from trend values for $\Delta_R$ and for $\Delta_P$ show significant contingency, we can assume that the variation in phosphorus content is connected only with variation in rock composition. In other words, we can check the hypothesis of two independent trends. The subtraction of the trend from the rock-range number permits us to place all values of observed figures into two classes. There are classes of positive (+) and negative (−) deviations from the trend surface. Now we can investigate the problem by analysis of four-fold contingency tables. It is possible to do this after subtraction of trends because we obtained independent observations. Before trend subtraction, all observations on the same random value were correlated, and that excluded the simple methods of estimation of significance of estimaters. The problem is similar to others investigated in previous work (Vistelius, 1971).

## CONTINGENCY TABLES FOR $\Delta_R$ AND $\Delta_P$ AND ITS ANALYSIS

Contingency between classes of rocks ($\Delta_R$), classes of phosphorus concentration ($\Delta_P$), and classes of geographic position of sampled points was investigated. The last factor was investigated in two ways—division of Colombia into southern and northern regions and division into western and eastern regions. Division for northern and southern regions was made along lat 6°45′ N. The boundary line dividing west from east extends from Santa Marta south to the junction of the Department of Antioquis, Caldas and Boyaca (lat 5°46′ N). From this point it runs to Honda on the Magdalena River, then west to the town of Ibague and sampling point TO-66 and south to Popayan and Pasto. The division of western and eastern regions is coincident only with a line of general strike of the Andean system (Fig. 1).

All data on $\Delta_R$ and $\Delta_P$ and their geographic position are listed in Table 1, and from Table 1 we obtain Table 3. There is one situation where $\Delta_4$ was equal to zero. We added this unit in each of the adjacent cells.

The data in Table 3 were analysed by information-theory statistics

**Table 3. Contingency Tables Between Rock-Range Numbers $\Delta_R$, Phosphorus Concentrations $\Delta_P$, and Geographic Position of Observation Points**

| Geographical division | | North | | | South | | | West | | | East | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Phosphorus concentrations | | − | + | Σ | − | + | Σ | − | + | Σ | − | + | Σ |
| Range numbers of rocks | − | 21 | 10 | 31 | 14 | 8 | 22 | 22 | 6 | 28 | 14 | 11 | 25 |
| | + | 7 | 13 | 20 | 10 | 21 | 31 | 7 | 16 | 23 | 10 | 18 | 28 |
| | Σ | 28 | 23 | 51 | 24 | 29 | 53 | 29 | 22 | 51 | 24 | 29 | 53 |

(Kulback, 1959). We used the following designations. Capital letters designate investigating characters and they are as follows. $\Gamma$ is a geographic division, $\Phi$ is the phosphorus concentration, and $\Pi$ is the rock type. Each character is divided into two grades. Grades are designated by small letters: $i$ are grades of $\Gamma$, $j$ are grades of $\Phi$, and $k$ are grades of $\Pi$. The sign $\times$ indicates that we want to check the hypothesis that factors to the left and to the right of this sign are independent. Say $\Phi \times \Pi$ indicates that we believe the values of $\Phi$ and $\Gamma$ are independent. Designation $\phi \times (\Gamma\Pi)$ indicates that we believe the $\phi$ is independent of $\Gamma$ and $\Pi$ simultaneously.

Designation $\Gamma \times \Pi/\Phi^{(-)}$ indicates that we believe that $\Gamma$ and $\Pi$ are mutually independent with the condition that the phosphorus concentration is below the trend surface (class "minus" or "$-$" of $\Delta_p$). Sign $=$ is synonymous with word "or" in an alternative. The alternative is located on the left-hand side of the equality. Systematic investigation indicates that we are able to check 19 hypotheses about different types of contingency. These hypotheses are the following.

$$\Gamma\Pi\Phi = \Gamma \times \Pi \times \phi$$

$$\Gamma(\phi\Pi) = \Gamma \times (\Phi\Pi) \qquad \Phi(\Gamma\Pi) = \Phi \times (\Gamma\Pi) \qquad \Pi(\Gamma\Phi) = \Pi \times (\Gamma\Phi)$$

$$\Gamma\Phi = \Gamma \times \phi \qquad \Gamma\Pi = \Gamma \times \Pi \qquad \phi\Pi = \Phi \times \Pi$$

$$\phi\Pi/N = (\Phi \times \Pi)/N \qquad \Phi\Pi/S = (\Phi \times \Pi)/S \qquad \phi\Pi/W = (\phi \times \Pi)/W$$

$$\Phi\Pi/E = (\Phi \times \Pi)/E$$

$$\Gamma^{(N-S)}\Phi/\Pi^{(+)} = (\Gamma^{(N-S)} \times \Phi)/\Pi^{(+)} \qquad \Gamma^{(N-S)}\Phi/\Pi^{(-)} = (\Gamma^{(N-S)} \times \Phi)/\Pi^{(-)}$$

$$\Gamma^{(W-E)}\Phi/\Pi^{(+)} = (\Gamma^{(W-E)} \times \Phi)/\Pi^{(+)} \qquad \Gamma^{(W-E)}\Phi/\Pi^{(-)} = (\Gamma^{(W-E)} \times \Phi)/\Pi^{(-)}$$

$$\Gamma^{(N-S)}\Pi/\Phi^{(+)} = (\Gamma^{(N-S)} \times \Pi)/\Phi^{(+)}$$

$$\Gamma^{(N-S)}\Pi/\Phi^{(-)} = (\Gamma^{(N-S)} \times \Pi)/\Phi^{(-)} \qquad \Gamma^{(W-E)}\Pi/\Phi^{(+)} = (\Gamma^{(W-E)} \times \Pi)/\Phi^{(+)}$$

$$\Gamma^{(W-E)}\Pi/\Phi^{(-)} = (\Gamma^{(W-E)} \times \Pi)/\Phi^{(-)}$$

It is known (Kulback, 1959, p. 124) that the hypothesis should be checked by the minimum discrimination information statistic $2\hat{I}$, which asymptotic distribution coincides with distribution of central $\chi^2$, in the situation if the hypothesis on the right-hand side of the equality is correct. For convenience of computation of $2\hat{I}$ for each hypothesis, the necessary formulas are given. These formulas were borrowed from Kulback and were written in expanded form suitable for calculation. Types of hypotheses are given before the formulas. The designations are as follows. $x$ is a number of observations; if $x$ has three subscripts ($x_{ijk}$), it is a number of observations in a cell of three-dimensional classification; if $x$ has two subscripts and a point ($x_{.jk}$) it is an indication of the number of observations in a cell of two-dimensional classification with ignorance of the factor grades which are indicated by a point. Similarly, $x$ with one subscript and two points ($x_{..k}$) indicates a number of observations in row or column (a cell of one-dimensional classification). In our example three-dimensional classification has eight cells, two-dimensional classification contains four cells, and one-dimensional classification is composed of two cells.

$N$ is a whole number of observations. It is obvious that

$$\sum_i \sum_j \sum_k x_{ijk} = \sum_i \sum_j x_{ij.} = \sum_i \sum_k x_{i.k} = \sum_j \sum_k x_{.jk} = \sum_i x_{i..} = \sum_j x_{.j.} = \sum_k x_{..k} = N$$

The formulas for computations are as follows, where $\nu$ is the degree of freedom, determined by formulas obtained by Kulback (1959).

$$XYZ = X \times Y \times Z \qquad X, Y, Z \in \{\Phi, \Pi, \Gamma\}$$

$$2\hat{I} = 2\Big(\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} x_{ijk} \ln x_{ijk} - \sum_{i=1}^{2} x_{i..} \ln x_{i..} - \sum_{j=1}^{2} x_{.j.} \ln x_{.j.} - \sum_{k=1}^{2} x_{..k} \ln x_{..k} + 2N \ln N\Big)$$

$\nu = 4$

$$XYZ = X \times (YZ)$$

$$2\hat{I} = 2\Big(\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} x_{ijk} \ln x_{ijk} - \sum_{i=1}^{2} x_{i..} \ln xi_{..} - \sum_{j=1}^{2} \sum_{k=1}^{2} x_{.jk} \ln x_{.jk} + N \ln N\Big)$$

$\nu = 3$

$$YZ = Y \times Z$$

$$2\hat{I} = 2\Big(\sum_{j=1}^{2} \sum_{k=1}^{2} x_{.jk} \ln x_{.jk} - \sum_{j=1}^{2} x_{.j.} \ln x_{.j.} - \sum_{k=1}^{2} x_{..k} \ln x_{..k} + N \ln N\Big)$$

$\nu = 1$

$$XY/Z^{(k)} = X/Z^{(k)} \times Y/Z^{(k)}$$

$$2\hat{I} = 2\Big(\sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij} \ln x_{ij} - \sum_{i=1}^{2} x_{i.k} \ln x_{i.k} - \sum_{j=1}^{2} x_{.jk} \ln x_{.jk} + x_{..k} \ln x_{..k}\Big)$$

$\nu = 1$

Insertion of $\Phi$, $\Gamma$, and $\Pi$ in the place of $X$, $Y$, and $Z$ permits us to check all possible hypotheses about contingency of investigated characters. As seen from the set of checking hypotheses, all of them can be divided into two classes: (1) general hypothesis, if we speak about characters independently of their partial values, and (2) conditioned hypotheses if we put conditions on one of the investigated characters. The general hypotheses clear up interrelations for all values of $\Phi$, $\Gamma$, and $\Pi$. Table 4 gives the result of checking general hypotheses.

**Table 4. Results of Checking General Hypotheses of Contingency Between Φ,Z, and Π**

| Hypothesis | Number of degrees of freedom $\upsilon$ | Approximate probability of error of first type $\alpha^a$ North–south | West–east |
|---|---|---|---|
| $\Gamma\Phi\pi = \Gamma \times \phi \times \Pi$ | 4 | 0.005 (15.40) | 0.005 (16.62) |
| $\Gamma(\Phi\Pi) = \Gamma \times (\Phi\pi)$ | 3 | 0.250 (4.64) | 0.250 (3.92) |
| $\Phi(\Gamma\Pi) = \Phi \times (\Gamma\Pi)$ | 3 | 0.010 (11.56) | 0.005 (16.60) |
| $\Pi(\phi\Gamma) = \Pi \times (\phi\Gamma)$ | 3 | 0.005 (14.46) | 0.005 (15.24) |
| $\Gamma\Pi = \Gamma \times \Pi$ | 1 | 0.010 (2.28) | 0.250 (0.62) |
| $\Gamma\Phi = \Gamma \times \phi$ | 1 | 0.250 (0.98) | 0.100 (1.38) |
| $\Phi\Pi = \Phi \times \Pi$ | 1 | 0.005 (12.70) | |

[a] Numbers in parentheses are values of $2\hat{I}$.

Table 4 gives the following information. If we accept the value of error of the first type equal to 0.05, as it is accepted in routine investigations, all hypotheses of independence for Π and Φ should be rejected on a high level. In other words, interrelations within $\phi\Pi$, $\Gamma\phi\Pi$, $\Phi(\Gamma\Pi)$, and $\Pi(\phi\Gamma)$ in which we clear up bounds between Π and Φ if the position of Γ is passive, should reject the hypothesis of independence, at a high level.

Situations $\Gamma(\phi\Pi)$, $\Gamma\Pi$, $\Gamma\Phi$ where geography is the character being checked indicate that we should accept the hypothesis of independence on a high level. There is only one situation (ΓΠ for north–south classes) where we should reject the hypothesis of independence. But it is a single situation and it can occur because our model is not ideal, and at the level of rejection the hypothesis of independence is smaller than in other situations. Thus Table 4 shows that it is better to accept the hypothesis where changes in $\Delta_R$ and $\Delta_P$ are dependent one on another, and there is no influence of geographic position of samples on these values.

Now, let us investigate contingency in fixed conditions or contingency for fixed geographic regions and types of deviations from the trend surface. Table

**Table 5. Conditioned Hypotheses of Interrelations Between Π, Φ, and Γ**

| Checking hypothesis | $v$ | $2\hat{I}$ | $\alpha$ |
|---|---|---|---|
| $\phi\Pi/N = (\Phi\times\Pi)/N$ | 1 | 5.34 | 0.025 |
| $\Phi\Pi/S = (\Phi\times\Pi)/S$ | 1 | 5.22 | 0.025 |
| $\phi\Pi/W = (\Phi\times\Pi)/W$ | 1 | 12.66 | 0.005 |
| $\Phi\Pi/E = (\Phi\times\Pi)/E$ | 1 | 2.66 | 0.250 |
| $\Gamma^{(N-S)}\Phi/\Pi^{(+)} = (\Gamma^{(N-S)}\times\Phi)/\Pi^{(+)}$ | 1 | 0.04 | 0.500 |
| $\Gamma^{(N-S)}\Phi/\Pi^{(-)} = (\Gamma^{(N-S)}\times\Phi)/\Pi^{(-)}$ | 1 | 0.12 | 0.750 |
| $\Gamma^{(W-E)}\Phi/\Pi^{(+)} = (\Gamma^{(W-E)}\times\Phi)/\Pi^{(+)}$ | 1 | 0.16 | 0.100 |
| $\Gamma^{(W-E)}\phi/\Pi^{(-)} = (\Gamma^{(W-E)}\times\Phi)/\Pi^{(-)}$ | 1 | 3.14 | 0.100 |
| $\Gamma^{(N-S)}\Pi/\Phi^{(+)} = (\Gamma^{(N-S)}\times\Pi)/\Phi^{(+)}$ | 1 | 1.40 | 0.250 |
| $\Gamma^{(N-S)}\Pi/\Phi^{(-)} = (\Gamma^{(N-S)}\times\Pi)/\Phi^{(-)}$ | 1 | 1.64 | 0.250 |
| $\Gamma^{(W-E)}\Pi/\Phi^{(+)} = (\Gamma^{(W-E)}\times\Pi)/\Phi^{(+)}$ | 1 | 1.66 | 0.500 |
| $\Gamma^{(W-E)}\Pi/\Phi^{(-)} = (\Gamma^{(W-E)}\times\Pi)/\Phi^{(-)}$ | 1 | 1.88 | 0.250 |

5 gives 12 conditioned hypotheses and data for their checking. In this table, $\Pi^{(N-S)}$ indicates that the division of the territory was made to the north and south, $\Pi^{(W-E)}$ indicates that the same division was made for the western and eastern regions. N after the inclined line is an indication for north, S indicates south, W is west, and E is east. $\Phi^{(+)}$ is an indication that we are dealing with positive deviations from trend surface for phosphorus. Other designations were used in a similar manner.

Table 5 shows that in all situations, the hypothesis of independence between geographic divisions and variations of $\Delta_R$ and $\Delta_P$ should be accepted on a rather high level of significance. Interrelations between $\Delta_R$ and $\Delta_P$ in three situations permit us to reject the hypothesis of independent variations on a high level. There is only one situation where independent variations between $\phi\Pi$ occur. It was for samples from the eastern region. This situation is impossible to explain by special reasons because our response model was introduced on a purely heuristic level. Thus, it is possible that in some region it will not fit the observed values and our observations will be dependent. If we compare this result of a conditioned hypothesis with deviation in interpretation for checking the general hypothesis, it is suspect that there is a general reason in both situations. For the general hypothesis, we had deviations for interrelations ΓΠ for west–east classes; we have deviations for the conditioned hypothesis ΦΠ with the fixed eastern region. Thus, there must be special behavior of rock-range numbers in the eastern region. For clarification of this conclusion, it would have been useful to make a special investigation, but we were restricted by a limited quantity of samples. It is clear, however, that interrelations are such that variations in rock composition can explain the origin of variation in phosphorus content. In other words, the trend in rock composition over Colombia within mountain systems is sufficient as an explanation for the origin of the trend in phosphorus concentration.

## SUMMARY

The content of phosphorus and the precise name of the rock (on the basis of careful microscopic examination) were determined in samples of granitic rocks, taken as uniformly as possible, and collected over the Andean part of Colombia.

Interpretation on origin of the variation in phosphorus concentration was accomplished by two different methods.

Trend surfaces for phosphorus concentration and for rock-range numbers were computed independently for samples at the same sampling points. Comparison of these trend surfaces by comparative trend analysis (COMTRENA) was made. COMTRENA showed that trends in phosphorus content of the rocks coincides with rock-range numbers which reflect variations in chemical composition.

After values of trend were subtracted from observed values of phosphorus concentration and rock-range numbers, the differences ($\Delta_R$ and $\Delta_P$) were investigated. Contingency tables in terms of information-theory statistics were used in this investigation. Computations showed that there is contingency between deviations in phosphorus content from the general trend in concentrations of this element, and deviations of rock-range numbers from their general trend.

The investigation revealed that variation in phosphorus content in granitic rocks can be explained sufficiently by changes in chemical composition of the whole rock. In other words, phosphorus content in granitic rocks reflects the main features of a whole-rock chemical composition.

## ACKNOWLEDGMENTS

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